AP* Calculus Question Type Analysis and Notes<br>Revised to include the 2012 Exam<br>By Lin McMullin

General note: AP Questions often test several diverse ideas or concepts in the same question. The type names are meant only as a guide and may refer to the form of the question, what it looks like at a glance. Within each type various ideas and concepts may and often are tested.

## Tables listing type by years and question number follow Topic 9 (page 7)

## Topic 1: Area - Volume Question

Given equations that define a region in the plane students are asked to find its area and the volume of the solid formed when the region is revolved around a line or used as a base of a solid with regular cross-sections. This standard application of the derivative has appeared every year.

If this appears on the calculator active section: It is expected that the definite integrals will be evaluated on a calculator. Students should write the definite integral with limits on their paper and put its value after it. It is not required to give the antiderivative and if students give an incorrect antiderivative they will lose credit even if the final answer is (somehow) correct.

Since 2009 this question appeared on the no calculator allowed section. This is because there is a calculator program available that will give the set-up and not just the answer. Expect the question to be on the no calculator section. (Good news is that the integrals will be easy or they will be set-up but do not integrate questions.)

## What students should know how to do:

- Find the intersection(s) of the graphs and use them as limits of integration (calculator equation solving). Write the equation followed by the solution; showing work is not required. Usually no credit until the solution is used in context.
- Find the area of the region between the graph and the $x$-axis or between two graphs.
- Find the volume when the region is revolved around a line by the disk/washer method. (Shell method is never necessary but is eligible for full credit if properly used).
- Find the volume of a solid with regular cross-sections whose base is the region between the curves. But see 2009 AB 4(b)
- Find the equation of a vertical line that divides the region in half (area or volume). This involves setting up and solving an integral equation where the limit is the variable for which the equation is solved.
- For BC only - find the area of a region bounded by polar curves.


## Topic 2: Particle moving on a line

These questions may give the position equation, the velocity equation or the acceleration equation along with an initial condition. Students may be asked about the motion of the particle: its direction, when it changes direction, its maximum position in one direction etc. Speed, the absolute value of velocity, is also a common topic.

The particle may be a "particle," a person, car etc. The position, velocity or acceleration may be given as an equation, a graph or a table. There are a lot of different things students may be asked to find. While particles don't really move in this way, the question is a versatile way to test a variety of calculus concepts.

The information may be given as an equation, a graph or in a table.

## What students should know how to do (AB and BC):

- Initial value differential equation problems: given the velocity or acceleration with initial condition(s) find the position or velocity.
- Distinguish between position at some time (displacement) and the distance traveled during the time.
o The total distance traveled is the definite integral of the absolute value of the rate of change (velocity): $\int_{a}^{b}|v(t)| d t$
o The net distance (displacement) is the definite integral of the rate of change (velocity): $\int_{a}^{b} v(t) d t$
o The final position is the initial position plus the definite integral of the rate of change from $x=a$ to $x=t: s(t)=s(a)+\int_{a}^{t} v(x) d x$ Notice that this is an accumulation function equation.
- Find the speed at a particular time. The speed is the absolute value of the velocity.
- Find average speed, velocity, or acceleration
- Determine if the speed is increasing or decreasing.

0 If at some time, the velocity and acceleration have the same sign then the speed is increasing.
o If they have different signs the speed is decreasing.

- Use difference quotient to approximate derivative
- Riemann sum approximations
- Units of measure
- Interpret meaning of integral in context of the problem


## Topic 3: Particle moving on a plane for BC (parametric/vector question).

On the BC exam particles often move in the plane. Their position is defined by two parametric equations $x=x(t)$ and $y=y(t)$. The velocity is the vector $\left(x^{\prime}(t), y^{\prime}(t)\right)$ and the acceleration is the vector $\left(x^{\prime \prime}(t), y^{\prime \prime}(t)\right)$. Any of these three may be given with initial conditions(s) and student may be asked to find the others.

## What students should know how to do:

- Initial value differential equation problems - given the velocity or acceleration with initial conditions, find the position and/or velocity.
- Use the definite integral for arc length to find the distance traveled.
- Find the speed at time $t$ : speed $=\sqrt{\left(x^{\prime}(t)\right)^{2}+\left(y^{\prime}(t)\right)^{2}}$
- Vectors are given in ordered pair form; answers may be in ordered pairs form or $\vec{i}-\vec{j}$ form.


## Topic 4: Interpreting Graphs

There are a variety of question types here. Students may be given an equation and asked for the location of extreme values, intervals where the function is increasing or decreasing, concavity, etc. Students may be given the graph of the derivative and asked the same kinds of things. They may be asked to find the value of the integral given the graph but no equation.

This may be a particle motion problem where the velocity is given as a graph.

## What students should know how to do:

- Reading information about the function from the graph of the derivative. This may be approached as a derivative techniques or antiderivative techniques.
- Find and justify extreme values ( $1^{\text {st }} \mathrm{DT}, 2^{\text {nd }} \mathrm{DT}$ ).
- Find and justify points of inflection.
- Write an equation of tangent line
- Evaluate Riemann sums from graphs only.
- FTC from graph areas.
- "Family" of functions": functions with a parameter;
- Functions defined by other functions


## Topic 5: Accumulation \& Rates

The integral of a rate of change gives the amount of change. The general form of the equation is $f(x)=f\left(x_{0}\right)+\int_{x_{0}}^{x} f^{\prime}(t) d t, x=x_{0}$ is the initial time, and $f\left(x_{0}\right)$ is the initial value. Since this is one of the main interpretations of the definite integral the concept may come up in a variety of situations.

## What students should know how to do?

- Understand the question. It is often not necessary to as much computation as it seems at first.
- The FTC may help differentiating $F$.
- Often these problems contain a lot of writing; be ready to read and apply.
- Explain the meaning of a derivative or definite integral or its value in terms of the context of the problem.
- In-out problems: 2 rates of change work together but in opposite directions.
- Max/min and inc/dec analysis.
- Explain the meaning of a definite integral in context. The explanation should include (1) what the integral gives, (2) the units and (3) an accounting of the limits of integration.


## Topic 6: Problems from information given in tables

Tables may be used to test a variety of ideas in calculus including analysis of functions, accumulation, position-velocity-acceleration, et al.

## What students should be able to do:

- Approximate the derivative using a difference quotient.
- Use Riemann sums (left, right, midpoint) or the Trapezoidal Rule to approximate the value of a definite integral using values in the table (typically with uneven subintervals).
- Average value and the MVT may appear
- Questions about the Rolle's theorem, MVT, IVT, etc.

Do not: Use a calculator to find a regression equation and then use that to answer parts of the question. (While finding them ids perfectly good mathematics, regression equations are not one of the four things students may do with their calculator.)

## Topic 7: Differential Equation Questions

Differential equations are tested every year. The actual solving of the differential equation is usually the main part of the problem, but it is accompanied by a question about its slope field or a tangent line approximation of some sort. BC students may also be asked to approximate using Euler's Method. Large parts of the BC questions are often suitable for AB students and contribute to the AB subscore of the BC exam..

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## What students should be able to do:

- Find the general solution of a differential equation using the method of separation of variables (this is the only method tested).
- Find a particular solution using the initial condition to evaluate the constant of integration - initial value problem (IVP)
- Understand that proposed solution of a differential equation is a function (not a number) and if it and its derivative is substituted into the given differential equation the resulting equation is true. This may be part of doing the problem even if solving the differential equation is not required (see 2002 BC 5(c))
- Growth-decay problems.
- Draw a slope field by hand.
- Sketch a particular solution on a (given) slope field.
- Interpret a slope field.
- For BC only: Use Euler’s Method to approximate a solution.
- For BC only: use the method of partial fractions to find the antiderivative after separating the variables.
- For BC only: understand the logistic growth model, its asymptotes, meaning, etc.


## Topic 8: Power Series (BC only)

Since some graphing calculator can produce Taylor Polynomials, this question appears on the no calculator allowed section. (Questions from 1995-1999 before the FR sections was split do not have anything a calculator could do. They are interesting and clever.)

## What students should be able to do:

- Find the Taylor (or Maclaurin) polynomial or series for a given function - usually 4 terms and the general term). This may be done by finding the various derivatives, or any other method such as substitution into a know series, long division, the formula for the sum of an infinite geometric series, integration, differentiation, etc.
- Know from memory the Maclaurin series for $\sin (x), \cos (x), e^{x}$, and $\frac{1}{1-x}$
- Find related series by substitution, differentiation, integration or by adapting one of those above.
- Find the radius of convergence (usually by using the Ratio test, or from a geometric series).
- Find the interval of convergence using the radius and checking the endpoints separately.
- Handle geometric series.
- Use the convergence tests.
- Find a high-order derivative from the coefficient of a term.
- Estimate the error bound of a Taylor or Maclaurin polynomial by using alternating series error bound or the Lagrange error bound.
- Do not claim that a function is equal to (=) its Taylor or Maclaurin polynomial; it is only approximately equal $(\approx)$. This could cost a point.


## Topic 9: Other Topics tested occasionally

## Implicitly defined relations and implicit differentiation

These questions may ask students to find the first or second derivative of an implicitly defined relation. Often the derivative is given and students are required to show that it is correct. (This is because without the correct derivative the rest of the question cannot be done.)

## What students should know how to do?

- Know how to find the first derivative of an implicit relation using the product rule, quotient rule, the chain rule, etc.
- Know how to find the second derivative, including substituting for the first derivative.
- Know how to evaluate the first and second derivative by substituting both coordinates of the point,
- Analyze the derivative to determine where the relation has horizontal and/or vertical tangents.
- Work with lines tangent to the relation.


## Related Rates

Derivatives are rates and when more than one variable is involved the relationships among the rates can be found by differentiating with respect to time. The time variable may not appear in the equations.

## What students should know how to do:

- Know how to differentiate using the product, quotient and chain rules.
- Set up and solve related rate problems.
- Interpret the answer.


## A Guide to the Free-response Questions

- GCA = Graphing calculator allowed - yes or no; GCA indicates that the question is in the graphing calculator allowed section of the exam; it does not indicate that a graphing calculator should or must be used.
- Comments are highlights, not a complete description
- Since often several topics are tested in the same question, some questions are listed more than once

| Topic 1: Area, volume $A B$ \& $B C$ |  |  |
| :---: | :---: | :---: |
| Year \& Question | GCA | Comments |
| 1998 AB1 | Yes | Line divides area or volume into equal parts |
| 1998 BC1 | Yes | Line divides volume into equal parts. Suitable for $A B$ |
| 1998 AB2-BC2 | Yes | (c) find other lane with same volume |
| 2000 AB1-BC1 | Yes | Find intersection, Regular cross-section |
| 2001 AB1 | Yes | Find intersection; 2 integrals needed. |
| 2002 AB1- BC1 | Yes | Includes max/min |
| 2002 AB1 B | Yes | Volume of revolution and regular cross section |
| 2002 BC3 B | Yes | Curve length Parts $a$ and $b$ suitable for $A B$ |
| 2003 AB1-BC1 | Yes | Regular cross-section |
| 2003 BC3 | Yes | Area and polar graphs, polar area |
| 2003 AB1 B | Yes | Also write tangent line equation |
| 2003 BC2 B | Yes | Same area with $d y$ and $d x$; also polar area |
| 2004 AB2-BC2 | Yes | Regular cross-section |
| 2004 AB1 B | Yes | Volume of revolution horizontal and vertical lines |
| 2004 AB6-BC6 B | No | Area in terms of parameter. |
| 2004 BC5 B | No | Average value (x2), Improper integral |
| 2005 AB1-BC1 | Yes | Curves define 2 regions. |
| 2005 AB1 B | Yes | Regular cross-section (semi-circles) |
| 2005 BC6 B | No | Improper integrals |
| 2006 AB1-BC1 | Yes | Rotations horizontal and vertical lines |

[^1] Board, which was not involved in the production of and does not endorse any of the material here.

| 2006 AB1-BC1 B | Yes | Area, volume rotation, area. |
| :--- | :--- | :--- |
| 2007 AB1-BC1 | Yes | Area, volume rotation, no graph given |
| 2007 AB1-BC1 B | Yes | Area, volume rotation |
| 2008 AB1-BC1 | Yes | Area, volume, regular cross section, (d) variable height |
| 2008 AB1 B | Yes | Area, volume, regular cross-section |
| 2008 AB3-BC3 B | Yes | From table: Area, Trap rule, flow volume. |
| 2008 BC 4 B | No | Area, volume perimeter |
| 2009 AB 4 | No | Area, cross section square and area given |
| 2009 AB 4 B | No* | Area, square cross section, revolve |
| 2009 BC 1 B | Yes | Area, semi-circle cross section perimeter |
| 2010 AB 4 - BC 4 | No* | Area, revolve, square cross section |
| 2010 AB 1-BC 1 B | Yes | Area, volume of revolution, square cross section |
| 2010 BC 4 B | No | Equation stem, analyze graph, max/min, asymptote not <br> mentioned but needs to be considered. Area with <br> improper integral. |
| 2011 AB 3 | No | Tangent line, area, volume of rotation |
| 2011 BC 3 | No | Perimeter, volume of revolution, related rate |
| 2011 AB 3 BC 3 B | No | Area, rectangular cross-section w/ variable height, <br> tangent line perpendicular to other graph. |
| 2012 AB 2 | Yes | Horizontal rectangles, square cross-section, divide area <br> in half with horizontal line. |

*Expect the Area-volume problem to be on the no calculator section from now on - there is a calculator program that will give students the set-up as well as the answer.

Topic 2: Motion on a line for AB and BC

| Year \& Question | GCA | Comments |
| :---: | :---: | :---: |
| 1998 AB3 | Yes | Given graph \& table: difference quotient, Riemann sum |
| 1999 AB 1 | Yes | Given v: analyze motion, find a, s and total distance |
| 2000 AB2-BC2 | Yes | Given graph \& equation |
| 2001 AB3-BC3 | Yes | Given a graph: analyze v, max/min, FTC |
| 2002 AB3 | Yes | Given v equation: Speed/velocity, Speed increasing? |
| 2002 AB3 B | Yes | Given v equation: Sketch graph, analyze motion |
| 2003 AB2 | Yes | Given $v$ graph: Speed increasing? Analyze motion |
| 2003 AB4 B | No | Given $v$ graph: Speed increasing? Analyze motion |
| 2004 AB3 | Yes | Given v graph: Speed increasing? Analyze motion |
| 2004 AB3-BC3 B | Yes | Given table of $v$ : explain integral, MVT, average $v$ |
| 2005 AB5-BC5 | No | Given $v$ graph: distance, $a$, average rate of change |
| 2005 AB3 B | Yes | Given $v$ equation: analyze motion. |
| 2006 AB4 | No | Table \& equation, average, Riemann sum, |
| 2006 AB6 B | No | From table, distance, FTC, analyze motion |
| 2007 AB 4 | No | Also find and use x " |
| 2007 AB 2 B | Yes | Acceleration, distance position |
| 2008 AB2-BC2 B | Yes | Given "speed": Accumulation, rate of change |
| 2009 AB1 BC 1 | Yes | Graph of $v$, find distance, describe trip |
| 2009 AB 6 B | No | Explain integral, Trap rule, acceleration. |
| 2010 AB-4 BC-4 B | No | Analyze motion, distance, accumulation, graph stem |
| 2010 AB 6 B | No | 2 particles, analyze motion, speeding up? |
| 2011 AB 1 | Yes | Speed, average velocity, distance, turning point |
| 2011 AB 5 BC 5 B | No | Table of v , find acceleration, interpret integral, left Riemann sum, MVT, related rate |
| 2012 AB 6 | No | Direction, total distance, acceleration and speed, final position |


| Topic 3: Particle motion in the plane; parametric equation, vectors for BC |  |  |
| :---: | :---: | :---: |
| Year \& Question | GCA | Comments |
| 1998 BC6 | Yes | Rectangular and parametric together, position, speed. |
| 1999 BC1 | Yes | Given position: graph, max/min, acceleration, speed |
| 2000 BC4 | No | Given $v$ : acceleration, position, slope, limit as $t \rightarrow \infty$ |
| 2001 BC1 | No | Given v: Distance, position by accumulation, speed |
| 2002 BC3 | Yes | Slopes, distance, max/min |
| 2002 BC1 B | Yes | Given position: draw graph, speed, distance |
| 2003 BC2 | Yes | Given $x^{\prime}$ but not $y^{\prime}$ : an atypical problem |
| 2003 BC4 B | No | Given position: when moving horizontally and vertically |
| 2004 BC3 | Yes | Given position: tangent lines, speed |
| 2004 BC1 B | Yes | Given velocity vector |
| 2005 BC1 B | Yes | Equation of tangent line; when is particle at rest? |
| 2006 AB4-BC4 | No | Table \& equation, average, Riemann sum, |
| 2006 BC3 | Yes | Parametric velocity, tangent line, limits, improper integral |
| 2006 BC2 B | Yes | Tangent line, acceleration speed, distance traveled, explain. |
| 2007 BC 2 B | Yes | Speed, distance, position, tangent line, acceleration |
| 2008 AB4-BC4 | No | From graph: Particle motion, inc/dec, speed |
| 2008 BC 1 B | Yes | Given velocity: acceleration, position, speed distance |
| 2009 BC 3 | Yes | Diver, max, total distance, |
| 2010 BC 3 | Yes | Speed, distance, interpret slopes, accumulation (x2) |
| 2010 BC 2 B | Yes | Vertical tangent, tangent line, speed, acceleration. |
| 2011 BC 1 | Yes | Speed, slope, position, total distance traveled |
| 2012 BC 2 | Yes | Slope, position, speed, acceleration, total distance |


| Topic 4: Information from graph or about the graph, family of functions, $f, f^{\prime}, f^{\prime \prime}$ |  |  |
| :---: | :---: | :---: |
| Year \& Question | GCA | Comments |
| 1998 AB2- BC2 | Yes | Limits, max/min, "family" |
| 1999 AB4 | Yes | From equation, 2-DT, function defined by other function |
| 1999 AB5-BC5 | Yes | From graph: Max/min, inc/dec, tangent line, |
| 2000 AB3 | Yes | From graph: $f, f^{\prime}, f^{\prime \prime}$, inc/dec, POI |
| 2001 AB3-BC3 | Yes | Particle motion (q.v.) setting |
| 2001 AB4-BC4 | No | From derivative equation: Max/min, tangent line, concavity |
| 2002 AB1 BC1 (c) | Yes | From Equation; Includes max/min area/volume |
| 2002 AB4-BC4 | No | Given $f$ ' graph: inc/dec, concavity, draw graph $f$ |
| 2002 BC5 (d) | No | Slope filed w/ draw solution, Euler, max/min |
| 2002 AB2 B | Yes | Given equation of $f^{\prime}$ : Max/min, accumulation |
| 2002 AB4-BC4 B | No | Given graph $f$ : values, inc/dec, concavity, Trap rule. |
| 2003 AB3 | Yes | Given table and graph: Diff quotient, Riemann sum, max/min, average value, accumulation |
| 2003 AB4-BC4 | No | Given graph $f^{\prime}$ : inc/dec, tangent line, FTC by area. |
| 2003 AB5 B | No | Given graph f: values, average values, MVT, POI |
| 2004 AB5 | No | Given graph $f$ : values, FTC, max/min, POI |
| 2004 AB2 B | Yes | From equation; Accumulation setting, max/min, inc/dec. |
| 2004 AB4-BC4 B | No | Given graph $f^{\prime}$ :inc/dec, product rule |
| 2005 AB4 | No | Tabular information about $f, f^{\prime}, f^{\prime \prime}$, FTC, max/min, draw graph, POI |
| 2005 AB4-BC4 B | No | Given graph $f^{\prime}$ : values, FTC from graph area, inc/dec. |
| 2006 AB3 | Yes | Periodic function, accumulation, tangent line |
| 2006 AB 2 B | Yes | Concavity, max/min, tangent line |
| 2006 AB4-BC4 B | Yes | From graph, derivative, area |
| 2007 AB2-BC2 | Yes | Rate in/out from equations and graph |
| 2007 BC4 | No | Tangent, concavity |
| 2007 AB 4 B | No | Given graph of $f^{\prime}$; max, concavity, POI, area |


| 2007 AB 6 | No | Find $f^{\prime}$ and $f^{\prime \prime}$, max/ min, POI from equation |
| :---: | :---: | :---: |
| 2008 AB4-BC4 | No | From graph: Particle motion, inc/dec, speed |
| 2008 AB 6 | NO | Write tangent line, critical points, POI, limit |
| 2008 BC 5 | NO | Given derivative: critical points, inc/dec, IVP |
| 2008 AB5-BC5 B | No | POI, max/min, average rate of change, MVT |
| 2009 AB1 BC1 | Yes | Particle motion |
| 2009 AB 6 | No | Values, point of inflection max/min |
| 2009 AB3 BC 3 B | Yes | Differentiability, roc, average roc, concavity, MVT |
| 2009 AB5 BC 5 B | No | Tangent, max/min, average roc |
| 2010 AB 3 | Yes | Accumulation, inc/dec max/min |
| 2010 AB 5 | No | $f-\mathrm{f}$ ' - $\mathrm{f}^{\prime \prime}$, inflection points, max/min |
| 2010 AB 2 B | Yes | f- f ' f ', horizontal tangent, concavity, tangent line from equation |
| 2010 AB-4 BC-4 B | No | Analyze motion, distance, accumulation, graph stem |
| 2010 BC 4 B | No | Equation stem, analyze graph, max/min, asymptote not mentioned but needs to be considered. Area with improper integral. |
| 2011 AB 4 | No | Max/min, POI, average rate of change, MVT |
| 2011 AB 4 B | No | Max,min, concavity, IVP or accumulation |
| 2011 AB 6 | No | Accumulation, critical point, FTC |
| 2011 AB 4 B | No | Given graph of $f$ work with related graphs: Average value, FTC, concavity, inc/dec, arc lenght |
| 2012 AB 3 - BC 3 | No | Graph stem, f-f' - f', extreme vaues, POI |

## Topic 5: Accumulation \& Rates

| Year \& Question | GCA | Comments |
| :---: | :---: | :---: |
| 1998 AB5-BC5 | Yes | Draw graph, average value, accumulation |
| 1999 AB3-BC3 | Yes | From table: Riemann sum, Rolle's theorem - MVT, average value |
| 2000 AB4 | No | 2 solution methods: accumulation, max/min. |
| 2002 AB2-BC2 | Yes | "Amusement Park:" In-out, amounts, max/min, values |
| 2002 AB2-BC2 B | Yes | Pollutants: max/min, amounts, accumulation |
| 2003 AB3 | Yes | From graph and table: difference quotient, max/min, Riemann sum, interpret integrals. |
| 2003 AB2 B | Yes | Heating oil: In-out, inc/dec, accumulation, max/min |
| 2004 AB1-BC1 | Yes | Traffic flow: average value, average rate of change, accumulation, inc/dec. |
| 2004 AB2 B | Yes | Mosquitoes: values, inc/dec, accumulation, max/min |
| 2005 AB2 | Yes | "Sandy Beach:" In-out, amounts, max/min, FTC |
| 2005 AB2-BC2 B | Yes | Water tank: In-out, amount, max/min, inc/dec |
| 2006 AB2-BC2 | Yes | "Thomasville:" Average value and application |
| 2007 AB2-BC2 | Yes | Water Tank; rate in/out |
| 2007 AB3-BC3 B | Yes | Wind chill |
| 2008 AB2-BC2 | Yes | "Concert tickets" from table. Derivative, Trap Rule, max/min, accumulation |
| 2008 AB 3 | Yes | Related Rate, max/min, accumulation |
| 2008 AB2-BC2 B | Yes | Given "speed": Accumulation, rate of change, Related Rate |
| 2009 AB 2 BC 2 | Yes | Rate and extensions |
| 2009 AB 3 | Yes | Cost and profit |
| 2009 AB 1 B | Yes | Rates, related rates |
| 2009 AB 2 B | Yes | Distance, interpret derivative, |
| 2010 AB 1 - BC 1 | Yes | Accumulation, in-out type, piecewise function |
| 2010 AB 2 - BC 2 | Yes | Difference quotient, Trap rule, explain, accumulation, max/min |
| 2010 AB 3 | Yes | Accumulation, from graph, graph analysis |
| 2010 BC 3 | Yes | Speed, distance, interpret slopes, accumulation (x2) |


| 2011 AB 2 BC 2 | Yes | Table, average value, Trap sum, accumulation |
| :--- | :--- | :--- |
| 2011 AB 1 BC 1 B | Yes | Accumulation, average rate of change, related rate, <br> IVT |
| 2011 AB 2 B | Yes | Continuity, average rate of change, accumulation. |
| 2012 AB 1 | Yes | Table stem, average ROC, interpret integrals, left- <br> Riemann sum, over/under estimate?, Accumulation <br> from equation. |


| Topic 6: Table |  |  |
| :--- | :--- | :--- |
| Year \& Question | GCA | Comments |
| 1998 AB3 | Yes | Given graph \& table: difference quotient, Riemann <br> sum |
| 1999 AB3-BC3 | Yes | From table: Riemann sum, Rolle - MVT, average <br> value |
| 2001 AB2-BC2 | Yes | Difference quotient, Trap rule, explain, average value |
| 2002 AB6 | No | Definite integral, tangent line, MVT, interpret, limits |
| 2003 AB3 | Yes | From graph and table: difference quotient, max/min, <br> Riemann sum, interpret integrals. |
| 2003 AB3 B | Yes | Average value, Riemann sum, explain, MVT |
| 2004 AB3-BC3 | Yes | Given table of v: explain integral, MVT, average $v$ |
| 2005 AB3-BC3 | Yes | Tabular information, FTC, max/min, draw graph, POI |
| 2005 AB4 | No | Tabular information about $f, f^{\prime}, f^{\prime \prime}$, FTC, max/min, <br> draw graph, POI |
| 2006 AB4 | No | Table \& equation, particle motion: average, Riemann <br> sum, |
| 2006 AB6 B | No | From table, distance, FTC, analyze motion |
| 2007 AB 3 | Yes | IVT, MVT, FTC, tangent line to inverse |
| 2007 AB5-BC5 | No | Relate rate, Riemann sum, rate of change, |
| 2008 AB2-BC2 | Yes | Difference Quotient, Trap Rule, max/min, <br> accumulation |
| 2008 BC 3 | Yes | Taylor polynomial, LaGrange error |
| 2008 AB 4 B | No | FTC, Chain rule (integral), tangent line, max/min |


| 2009 AB 5 BC 5 | No | Derivative, integral, left Riemann sum, tangent line <br> secant line and error analysis |
| :--- | :--- | :--- |
| 2009 AB 6 B | No | Particle motion, explain integral, Trap rule, <br> acceleration. |
| 2010 AB 2-BC 2 | Yes | Difference quotient, Trap rule, explain, accumulation, <br> max/min |
| 2010 AB-3 BC-3 B | Yes | Midpoint Riemann sum, accumulation, Related rate |
| 2011 AB 2 BC 2 | Yes | Table, average value, Trap sum, accumulation |
| 2011 AB 5 BC 5 B | No | Table of v, find acceleration, interpret integral, left <br> Riemann sum, MVT, related rate |
| 2012 AB 1 | Yes | Table stem, average ROC, interpret integrals, left- <br> Riemann sum, over/under estimate?, Accumulation <br> from equation. |
| 2012 BC 4 | No | 4 Approximations: Linear, midpoint-Riemann sum, <br> Euler's method, Taylor polynomial |

Topic 7: Differential Equations

| Year \& Question | GCA | Comments |
| :--- | :--- | :--- |
| 1998 AB4 | Yes | IVP, equation of tangent line |
| 1998 BC4 | Yes | IVP field, Euler, solve |
| 1999 BC6 | Yes | Tangent line with approximation, Euler, FTC with <br> "impossible integrand" in (c) |
| 2000 AB6 | No | IVP, find domain and range of solution. |
| 2000 BC6 | No | Slope field with analysis, IVP, range |
| 2001 AB6 | No | Second derivative by implicit differentiation of $y^{\prime}$, IVP |
| 2001 BC5 | No | Improper integral, Euler, IVP |
| 2002 BC5 | No | Draw solution on given slope filed, Euler, max/min, <br> check solution, 2-DT, |
| 2002 AB5-BC5 B | No | 2-DT, IVP |
| 2003 AB5-BC5 | No | Related Rate, IVP |
| 2003 BC6 (c) | No | From power series check solution of differential <br> equation |
| 2003 AB 6 B | No | Second derivative by implicit differentiation of $y^{\prime}$, IVP |
| 2004 AB6 | No | Draw slope field, generalize, IVP |


| 2004 BC5 | No | Logistics Diff Eq, limits from $y^{\prime}$ and $y$, interpret, IVP |
| :---: | :---: | :---: |
| 2004 AB5 B | No | Draw slope field, generalize, IVP |
| 2005 AB6 | No | Draw slope field, tangent line approximation, IVP |
| 2005 BC4 | No | Draw slope field and sketch solution on it; max/min, Euler, implicit differentiation for $y^{\prime \prime}$ and analyze. |
| 2005 AB6 B | No | Draw slope field, tangent line, solve |
| 2006 AB 5 | No | Draw slope field, solve, domain |
| 2006 BC 5 | No | Implicit y", write Taylor Poly, Euler |
| 2006 AB 5 B | No | Draw slope field, solve IVP |
| 2006 BC5 B | No | Solve IVP, Analyze logistic DE w/o solving. |
| (2007 AB 4 part b) | No | Similar to some DE work. |
| 2007 AB5 B | No | Slope field, no solution required, find coefficients, |
| 2007 BC 5 B | No | Find coefficients, Euler, |
| 2008 AB 5 | No | Slope field, solve IVP, limit |
| 2008 BC 6 | No | Draw solution on slope field, IVP, Euler, Taylor, range |
| 2008 BC 5 | NO | Given derivative: critical points, inc/dec, IVP |
| 2009 BC 4 | No | Solve, Euler, Taylor polynomial |
| 2010 AB 6 | No | Tan line, tan line approximation, concavity, solve DEq |
| 2010 BC 5 | No | Euler's method, L'Hôpital's Rule, Solve DEq |
| 2010 AB 5 B | No | Slope field, solve DEq |
| 2010 BC 6 B | No | Ratio test, half-open interval, use to verify DEq solution |
| 2011 AB 5 BC 5 | No | Tangent line approx., solve (NB: Median score = 0) |
| 2011 AB 4 | No | IVP in part (d) |
| 2012 AB 5 | No | Interpret derivative at point, second derivative (implicit) and discuss graph, solve separable IVP |

[^2]| Topic 8: Power Series BC only |  |  |
| :---: | :---: | :---: |
| Year \& Question | GCA | Comments |
| 1998 BC3 | Yes | Write Taylor, new series by substitution, and by integration, analyze. CAS no help |
| 1999 BC4 | Yes | Write Taylor series, Lagrange error bound, max/min CAS no help |
| 2000 BC3 | No | Write series, radius of convergence, Alternating series error bound |
| 2001 BC6 | No | Interval of convergence, limit, integrate, Geometric series sum |
| 2002 BC6 | No | Interval of convergence, differentiate. |
| 2002 BC6 | No | Substitute, series convergence |
| 2003 BC6 | No | Max/min 2-DT, Alternating series error; check solution of differential equation |
| 2003 BC6 B | No | Write series, radius of convergence, interval of convergence |
| 2004 BC6 | No | Write series, Find high-order coefficient, Lagrange error bound, integrate |
| 2004 BC2 B | Yes | Find derivatives, analyze, Lagrange error bound |
| 2005 BC6 | No | Write series, find general coefficient, interval of convergence |
| 2005 BC3 B | Yes | Given $f^{(n)}$, max/min 2-DT, write series, find radius of convergence. |
| 2006 BC 5 | No | Implicit y", write Taylor Poly, Euler |
| 2006 BC 6 | No | Interval w/ endpoints, $y^{\prime}$ and $y^{\prime \prime}$ from coefficients max/min |
| 2006 BC6 B | No | Differentiate, integrate, alternating series error bound. |
| 2007 BC 6 | No | Write terms, find limit using series, estimate using series, alternating series test |
| 2007 BC 6 B | No | Find terms, integrate, find coefficients, recognize know series. |
| 2008 BC 3 | Yes | From Table: Taylor polynomial, LaGrange error |
| 2008 BC 6 | No | Draw solution on slope field, IVP, Euler, Taylor, range |
| 2008 BC 6 B | No | Maclaurin series, integrate, error |
| 2009 BC 6 | No | Taylor, Ratio test, points of inflection |
| 2009 BC 6 B | No | Geometric, sum, integral, substitute. |
| 2010 BC 6 | NO | Write series (x2) max/min, Alternating series erro bound. |


| 2010 BC 6 B | No | Ratio test, half-open interval, use to verify DEq <br> solution |
| :--- | :--- | :--- |
| 2011 BC 6 | No | Sin(x) and cos(x) series, Lagrange error bound from <br> graph. |
| 2011 BC 6 | No | Write general term, interval of convergence given <br> radius, approximation, alternating series error <br> bound. |
| 2012 BC 4 | No | Table stem: 4 Approximations: Linear, midpoint- <br> Riemann sum, Euler's method, Taylor polynomimal |
| 2012 BC 6 | No | Find interval of convergence (ratio test and <br> endpoints), Alternating series error bound, write <br> terms |

## Topic 9 Miscellaneous

| Year \& Question | GCA | Comments |
| :--- | :--- | :--- |
| 1998 AB6 | Yes | Implicit diff, analyze implicit relation, tangent lines |
| 1999 AB6 | Yes | Related Rate |
| 2000 AB5-BC5 | No | Implicit diff, analyze implicit relation, tangent lines |
| 2001 AB5 | No | Unique graph problem, FTC |
| 2002 AB5 | No | Related Rate, units of measure |
| 2002 AB6 B | No | Related Rate |
| 2003 AB5-BC5 | No | Related Rate w/ solve differential equation IVP |
| 2003 AB6 | No | Continuity, average value, parameters |
| 2003 BC3 | Yes | Area and polar graphs, polar area |
| B 2003 BC3 | Yes | Includes polar area |
| 2004 AB4-BC4 | No | Implicit diff, analyze, second derivative |
| 2004 AB6-BC6 B | No | Use integral with parameter |
| 2005 BC2 | Yes | Polar Graph, max/min, interpret dr / d $\theta$ |
| 2005 AB5-BC5 B | No | Implicit diff, analyze implicit relation |
| 2006 AB6 | No | Differentiation without functions. |
| 2006 BC3 | Yes | Parametric velocity, tangent line, limits, improper <br> integral |
| 2006 BC 5 | No | Implicit y", write Taylor Poly, Euler |


| 2006 AB3-BC3 B | Yes | Graph investigation |
| :---: | :---: | :---: |
| 2007 BC 3 | Yes | Polar Equation, area, interpret derivatives |
| 2007 AB 3 | Yes | IVT, MVT, FTC, tangent line to inverse |
| 2007 AB 6 B | No | IVP, MVT, POI from generic function |
| 2008 AB2 | Yes | Related Rate, max/min, accumulation |
| 2008 AB5-BC5 B | No | POI, max/min, average rate of change, MVT |
| 2008 AB 2 B | Yes | Accumulation, Related Rate |
| 2008 AB 6 B | No | Implicit differentiation, tangent, analyze |
| 2009 AB 1 B | Yes | Related rate |
| 2009 BC 4 | No | Polar, area, derivative, tangent line |
| 2010 AB-3 BC-3 B | Yes | Midpoint Riemann sum, accumulation, Related rate |
| 2010 BC 4 B | No | Equation stem, analyze graph, max/min, asymptote not mentioned but needs to be considered. Area with improper integral. |
| 2011 AB 6 | No | Continuity, piecewise function, average value. |
| 2011 BC 3 | No | Perimeter, volume of revolution, related rate |
| 2011 AB 1 BC 1 B | Yes | Accumulation, average rate of change, related rate, IVT |
| 2011 AB 2 B | Yes | Continuity, average rate of change, accumulation. |
| 2011 AB 5 BC 5 B | No | Table of v , find acceleration, interpret integral, left Riemann sum, MVT, related rate |
| 2011 BC 2 B | Yes | Area, graph, find and interpret dy/dt |
| 2012 AB 4 | No | Tangent line, continuity of piecewise function, u-sub integration by hand, |

MVT:1999 AB3, 2002 AB6, 2003B AB3, 2004B AB3, 2005 AB3, 2006B AB6, 2007B AB6, 2007 AB3, 2008 AB2, 2009B AB3/BC3,


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